Varianta 022

Subiectul I

a) $|z|=\sqrt{5}$ .b) $\begin{vmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix}=0$ .c) $\dfrac{\sqrt{2}}{2}+1$ .d) $S=\dfrac{1}{2}|\Delta|=2, \Delta=\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{vmatrix}$. e) c=2. f) M(1,2).

Subiectul II

1) a) b–a=1$\in$ **N**. b) 0. c) A=$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, B=$\begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix}$. d) $f(-1)=f(1) \Rightarrow f$ nu este injectivă.

e) $2^4=16$.

2) a) $f'(x)=3x^2+1>0$, $\forall x \in \mathbf{R} \Rightarrow f$ este strict crescătoare.

b) $\lim\limits_{x\to 0}\left[(1+x)^{\frac{1}{x}}\right]^{\frac{1}{2}}=e^{\frac{1}{2}}=\sqrt{e}$. c) $y-f(1)=f'(1)\cdot(x-1) \Leftrightarrow y-2x+1=0$.

d) $\displaystyle\int_0^1 \frac{arctgx}{1+x^2}dx=\frac{\pi^2}{32}$.

e) $f'(x)=\dfrac{-x^2+1}{(x^2+1)^2}$, $A\left(-1,-\dfrac{1}{2}\right)$ punct de minim local şi $B\left(1,\dfrac{1}{2}\right)$ punct de maxim local.

Subiectul III

a) Se verifică prin calcul direct.

b) $f(x)=c_n\cdot X^n+c_{n-1}\cdot X^{n-1}+\ldots+c_1\cdot X+c_0$

$f(a)-f(b)=c_n(a^n-b^n)+c_{n-1}(a^{n-1}-b^{n-1})+\ldots+c_1(a-b)=$

$=c_n(a-b)(a^{n-1}+a^{n-2}b+\ldots+ab^{n-2}+b^{n-1})+c_{n-1}(a-b)(a^{n-2}+a^{n-3}b+\ldots+ab^{n-3}+b^{n-2})+\ldots+$

$+c_2(a-b)(a+b)+c_1(a-b) \Rightarrow (a-b) \mid (f(a)-f(b))$.

c) Presupunem ca $f(3)=5$. Conform punctului c) rezulta ca $(3-0)|(f(3)-f(0)) \Rightarrow 3|4$, fals.

d) Presupunem că f are rădăcina $k\in\mathbf{Z} \Rightarrow f(k)=0$. Pentru k par$\Rightarrow$

$(f(k)-f(2006)) \vdots (k-2006) \vdots 2 \Rightarrow -2007 \vdots 2$ fals

Pentru k impar $\Rightarrow (f(k)-f(2007)) \vdots (k-2007) \vdots 2 \Rightarrow -1003 \vdots 2$ fals.

e) $h(X)=a+b-X$.

f) $g(a)-g(b)=k_1(a-b)=b-c$; $g(b)-g(c)=k_2(b-c)=c-a$;

$g(c)-g(a)=k_3(c-a)=a-b$. $k_1,k_2,k_3\in\mathbf{Z}$. Prin înmulţire $\Rightarrow k_1\cdot k_2\cdot k_3=1$. Dacă $k_1=-1 \Rightarrow$ a–b=c–b $\Rightarrow$ a=c contradicţie cu a, b, c distincte. Dacă $k_2=1 \Rightarrow$ b–c=c–a $\Rightarrow$

c=$\dfrac{a+b}{2}$ şi analog. b=$\dfrac{a+c}{2}$ de unde rezulta b=c, contradicţie.

g) n=2 avem un polinom conform punctului e), iar pentru n=3 nu avem polinoame conform demonstrației de la f). La fel se demonstrează şi pentru n>3 ca nu exista polinoame singura valoare este n=2 .

Subiectul IV

a) B(2,2)= $\int_0^1 x\cdot(1-x)dx=\int_0^1 xdx-\int_0^1 x^2dx=\left.\frac{x^2}{2}\right|_0^1-\left.\frac{x^3}{3}\right|_0^1=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$.

b) B(a,2)= $\int_0^1 x^{a-1}(1-x)dx=\left.\frac{x^a}{a}\right|_0^1-\left.\frac{x^{a+1}}{a+1}\right|_0^1=\frac{1}{a}-\frac{1}{a+1}=\frac{1}{a(a+1)}$.

c) $B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx=-\int_1^0(1-t)^{a-1}\cdot t^{b-1}dt=\int_0^1(1-x)^{a-1}\cdot x^{b-1}dx=B(b,a)$.

d) $B(2,2)+B(2,3)+\ldots+B(2,n)=\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n(n+1)}=\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{n}-\frac{1}{n+1}$

$=\frac{1}{2}-\frac{1}{n+1}\to\frac{1}{2}$.

e) Se integrează prin părți.

f) $B(m,n)=\frac{n-1}{m+n-1}B(m,n-1)=\frac{n-1}{m+n-1}\cdot\frac{n-2}{m+n-2}\cdot B(m,n-2)=\ldots=$

$=\frac{(n-1)\cdot(n-2)\cdot\ldots\cdot 2\cdot 1}{(m+n-1)\cdot(m+n-2)\cdot\ldots\cdot(m+1)}\cdot B(m,1)=\frac{(n-1)!}{(m+n-1)\cdot(m+n-2)\cdot\ldots\cdot(m+1)}\cdot B(1,m)=$

$=\ldots=\frac{(n-1)!}{(m+n-1)\cdot(m+n-2)\cdot\ldots\cdot(m+1)}\cdot\frac{m-1}{1+m-1}\cdot\frac{m-2}{m-1}\cdot\ldots\cdot\frac{2}{3}\cdot\frac{1}{2}=\frac{(n-1)!\cdot(m-1)!}{(m+n-1)!}$.

g) Fie $x_n=B(n,n)$. Avem $\lim_{n\to\infty}\frac{x_{n+1}}{x_n}=\frac{1}{4}$ de unde rezulta ca $\lim_{n\to\infty}B(n,n)=0$.